Bohm's Mysterious 'Quantum Force' and 'Active Information': Alternative Interpretation and Statistical Properties

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An alternative interpretation to Bohm's 'quantum force' and 'active information' is proposed. Numerical evidence is presented, which suggests that the time series of Bohm's 'quantum force' evaluated at the Bohmian position for non-stationary quantum states are typically non-Gaussian stable distributed with a flat power spectrum in classically chaotic Hamitonian systems. An important implication of these statistical properties is briefly mentioned.

Key words: Bohm's Quantum Force; Active Information; Stable Distribution; Power Spectrum; Chaotic System.

According to Bohm's [1, 2] causal or ontological interpretation of quantum theory, which reproduces precisely all the predictions of the Copenhagen interpretation, matter has a well-defined trajectory, independent of observers. The motion of a particle is governed by a first-order differential equation for the position $x(t)$, or equivalently, by a second-order differential equation that has the form of Newton's second law of motion [1, 2]

$$\frac{\mathrm{d}\left(m\dot{x}(t)\right)}{\mathrm{d}\,t} = \left(-\nabla V(x,t) - \nabla Q(x,t)\right)\big|_{x(t)}. \quad (1)$$

Bohm [1] initially viewed the second term on the right-hand-side of (1), i. e., $-\nabla Q(x,t)$, as a physical force, which he called a 'quantum force', that acts on the particle of mass m at each position $x(t)$ in addition to the external classical force $-\nabla V(x,t)$. The quantum force is derived from the 'quantum potential'

$$Q(x,t) = -\frac{\hbar^2}{2m}\,\frac{\nabla^2 R(x,t)}{R(x,t)}, \quad (2)$$

which is determined by the amplitude $R(x,t)$ of the quantum wavefunction that time-evolves according to the time-dependent Schrödinger equation. Moreover, Bohm [1] regarded the quantum force as a fundamental physical force in addition to the four known ones. One of the mysterious features of this interpretation is that, unlike the other four fundamental forces, the quantum force does not have a source analogous to mass, charge, etc. [2, 3].

Bohm and Hiley [2] later proposed that $-\nabla Q(x,t)$ in (1) is 'not pushing or pulling the particle mechanically' as Bohm [1] had thought originally. Instead of a force, they [2] suggested that $-\nabla Q(x,t)$ represents 'active information' that guides the particle or informs the particle how to move like radar signals that guide a ship with an automatic pilot. Furthermore, they suggested that particles might have a 'complex and subtle inner structure' [2] that can somehow read the active information. This interpretation of $-\nabla Q(x,t)$ as active information is even more mysterious than the original interpretation of a fundamental physical force: in particular, what is the nature of the inner structure of the particle and how does it read the active information?

Here we propose an alternative interpretation of $-\nabla Q(x,t)$ to Bohm's quantum force and active information. First, we note that the external classical potential $V(x,t)$ determines the time-dependent wavefunction $\psi(x,t)$ through the time-dependent Schrö-

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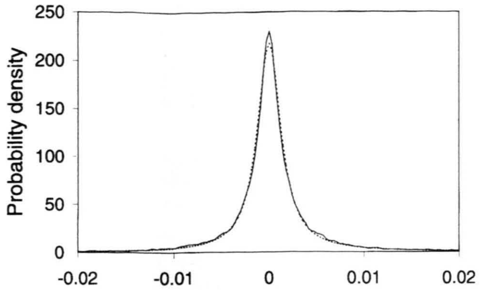

Fig. 1. Probability density function of a typical time series of $-\nabla Q(x,t)$ evaluated at the Bohmian position for the periodically delta-kicked plane pendulum: estimated data density (solid line) from 25001 data values, and fitted stable density (dotted line). The fitted stable density is essentially symmetric (skewness = 0.015) but highly non-Gaussian (characteristic exponent = 1.04) with scale = $1.4 \cdot 10^{-3}$ and location = $9.3 \cdot 10^{-6}$. All stable density parameters are given in the S^0-parametrization [5].

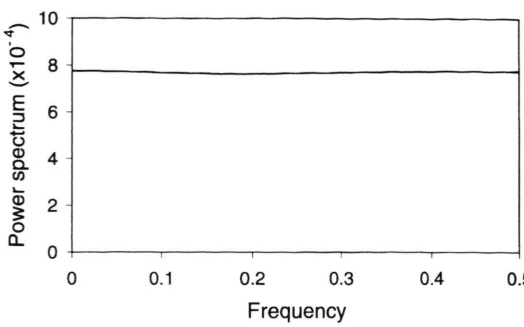

Fig. 2. Power spectrum of the time series data set of Fig. 1 versus frequency (in arbitrary unit) up to the Nyquist frequency of 0.5. Three poles were used to calculate the spectrum.

dinger equation. In turn, $\psi(x,t)$ determines $Q(x,t)$ via its amplitude $R(x,t)$ (see (2)). Thus $-\nabla Q(x,t)$ is *essentially* derived from the external *classical* potential $V(x,t)$. The common origin of $-\nabla Q(x,t)$ and the external classical force $-\nabla V(x,t)$ from the same classical potential suggests that $-\nabla Q(x,t)$ should not be interpreted as a separate force that acts on the particle in addition to the classical one. Instead, it suggests the interpretation of $-\nabla Q(x,t)$ as a *correction* to $-\nabla V(x,t)$ that produces the *actual* external classical force experienced by the particle, i.e., $-\nabla V(x,t) - \nabla Q(x,t)$ on the right-hand-side of (1).

Our alternative interpretation of $-\nabla Q(x,t)$ above to Bohm's is not meant to be a definitive one at this stage, but rather mainly to show that it is neither necessary to invoke a mysterious fundamental force without a source nor a mysterious active information that somehow guides a particle. Nevertheless, our alternative interpretation might serve as a more useful starting point for further understanding of Bohm's equation of motion (1).

Besides attempting to understand the meaning of $-\nabla Q(x,t)$, we have also studied its statistical properties. Our studies suggest that the time series of $-\nabla Q(x,t)$, evaluated at the Bohmian position for non-stationary quantum states, exhibits *typical* or *generic* statistical properties in classically chaotic Hamiltonian systems but not in non-chaotic ones.

In particular, our numerical findings suggest that, in chaotic systems, the time series are typically non-Gaussian stable [4] distributed with a flat power spectrum. Figure 1 shows a typical estimated data density (estimated using a Gaussian kernel [5]) together with a well-fitted stable density (fitted using a maximum likelihood method [5]) for a well-known prototypical classically chaotic Hamiltonian system: the periodically delta-kicked plane pendulum [6]. In this example, the system parameters are $a = mLgT = 0.005$ and $b = T/mL^2 = 50$ where the kicking period $T = 1$ (m and L are, respectively, the mass and length of the pendulum; g is the acceleration due to gravity). The initial wave function is an eigenstate of the free rotor $\exp(in\theta)/\sqrt{2\pi}$ where $n = 3142$. The initial Bohmian angle is π. We chose $\hbar = 0.0001$ for computational ease. All quantities with dimensions are in arbitrary units. Figure 2 shows the flat power spectrum, calculated using the maximum entropy (all poles) method [7], of the time series (sampling interval is equal to the kicking period) whose probability density is shown in Figure 1. Calculation details and other examples are given in a forthcoming publication [8], where a new method of calculating the quantum probability density of a particle's position implied by these statistical properties of the time series of $-\nabla Q(x,t)$ evaluated at the Bohmian position is described.

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[4] The class of stable distributions which allows for heavy tails and skewness is characterized by four parameters: characteristic exponent $\alpha \in (0,2]$, skewness $\beta \in [-1,1]$, scale $\gamma \in (0,\infty)$, and location $\delta \in (-\infty,\infty)$. The Gaussian ($\alpha = 2$) is the only member of this class of distributions with a finite variance; all other members ($0 < \alpha < 2$) have infinite variance and heavier density tails than the Gaussian (the tails are heavier for smaller*α). The distribution is symmetric if $\beta = 0$. If $\beta > 0$ ($\beta < 0$), the distribution is skewed to the right (left), i. e., the right (left) tail is longer. The degree of skewness is larger for larger $|\beta|$. For further details, see for example G. Samorodnitsky and M. S. Taqqu, Stable Non-Gaussian Random Processes, Chapman and Hall, New York 1994.

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